The Sphynx's new riddle: How to relate the canonical formula of myth to quantum interaction

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Abstract. We introduce Claude Lévi Strauss' canonical formula (CF), an attempt to rigorously formalise the general narrative structure of myth. This formula utilises the Klein group as its basis, but a recent work draws attention to its natural quaternion form, which opens up the possibility that it may require a quantum inspired interpretation. We present the CF in a form that can be understood by a non-anthropological audience, using the formalisation of a key myth (that of Adonis) to draw attention to its mathematical structure. The future potential formalisation of mythological structure within a quantum inspired framework is proposed and discussed, with a probabilistic interpretation further generalising the formula.

1 Introduction

Every society has its myths, and these show many similarities across societies which are themselves markedly different. Thus, a wide range of peoples have a "trickster" character; American Indians have Coyote, the Norse Loki, Africans Anansi, Christians Satan etc. and these characters share universal features despite their very different shapes and backgrounds. They even take similar roles in the mythological cycles that they participate in. Thus, many tricksters are central to creation myths, and equally they participate in the end of the world cycles. Such apparent universalities have led many [1] to wonder if there might be a general pattern to the myths of the world, or even a universal structure or essence [2].

One of the more mathematically oriented attempts to describe such a universal structure was first proposed by Claude Lévi-Strauss [3] in 1955. His *canonical formula* (CF) takes a structural approach to the analysis of myth, utilising mutually opposite value sets encoded in bundles of relations to consider the form that a myth takes as its storyline progresses. Intriguingly, this formula has its roots in group theory [4], which suggests that it might fit within a quantum inspired framework. However, the CF has been dividing anthropologists over the past sixty years and holds a somewhat enigmatic status within that community [5]. We believe that this controversy arises from the lack of a consistent interpretative framework from which to understand the CF, which in itself results in no universal understanding of the proper methodology for using the apparatus that Lévi-Strauss created. However, hope lies in the group theoretic formulation of the CF, and this paper is an attempt to propose that the framework of Quantum Interaction (QI) could provide an viable new way forward.

Here, we shall introduce the CF to the QI community, showing that it has strong parallels with many features that can be found in quantum inspired models, and so could provide a new exciting avenue of research. Section 2 gives a brief overview of the CF, and section 2.1 introduces the reader to its usage through the formalisation of a myth (that of Adonis) within the framework. Section 2.1 also explains how to relate the CF to running text by a syntagmatic reading. Section 2.2 discusses the difference between narrative formulae and the CF. Section 2.3 outlines a particular scenario which can generate literally hundreds of narrative formulae, among them 31 equivalents of the CF. Finally, section 3 cites a quaternion interpretation of such formulae with further implications in quantum theory. Section 5 sums up our conclusions.

2 The canonical formula

André Weil first wrote the CF as a formula of unfolding, formalizing it by means of group theory [6]. As far as this formula is understood, it describes plot (storyline) development in myths [3] or topic evolution in mythologies [7], encoded as a double transformation of four compound arguments in specific relation to one another:

$$F_x(a) : F_y(b) :: F_x(b) : F_{a^{-1}}(y). \tag{1}$$

Each of these four arguments consist of a term variable (a and b), and a function variable (x and y). The form of this equation requires some explanation, but we caution the reader that (1) has been the subject of ongoing and unresolved debate ever since Lévi-Strauss first proposed it. In what follows we shall closely follow the interpretation proposed by Morava [8], as this mathematically rigorous form will provide the basis for our claim that equation (1) can be understood within a quantum-like perspective.

In Morava's rendering, a number of different authors have suggested that (1) describes a *transformation*, which, for a sufficiently large and coherent body of myths, identifies

> *characters a, b and functions x, y, such that the mythical system defines a transformation which sends a to b, y to a^{-1} and b to y, while leaving x invariant.* [8, p.3]

This explanation leaves us with an interesting possibility for generating a mathematical description of myths, that is, the CF describes a *structural relationship* between a set of narrative terms and their transmutative relationships, however, the choice of what concepts these terms and relationships should apply to is left rather open and ill-defined. Intriguingly, at the root of the CF is a Klein group of four elements, e.g. x, $1/x$, $-x$, $-1/x$ [4], applied to one of the two narrative terms a and b or one of the two relations x and y, however Morava makes a convincing argument that the quaternion group of order eight is the correct mathematical structure to adequately represent Lévi-Strauss' conceptualisation, a point that we shall return to in section 3. For now, we shall leave this important point aside, attempting instead to illustrate the key features of the CF with reference to an example.

2.1 Applying the canonical formula

The CF (equation (1)) describes the relationship between syntagms, i.e. short sentences with condensed content which sum up parts of a myth, leaving one with a considerable amount of freedom when attempting to apply it to a narrative plot. This is a problem that becomes even more extreme when it is acknowledged that many different structural forms of the CF are consistent with the group that it specifies (see section 2.3). This complexity aside, application of the CF to a narrative consists of finding a consistent mapping of the objects and relations according to the structural relationship exemplified by (1), or one of its 32 alternatives (see the discussion in section 2.3, and the further generalization in section 3).

This is no easy task. It requires both the identification of suitable mythological narratives, and then the mapping of their components into a form mandated by (1), practically filling in placeholders in prespecified relationships to one another with fitting syntagmatic content. We shall illustrate this process with reference to an example myth, that of the Ancient Greek story of Adonis, which runs as follows [9, sections 14–16][3]:

> *Panyasis says that he was a son of Thias, king of Assyria, who had a daughter Smyrna. In consequence of the wrath of Aphrodite, for she did not honor the goddess, this Smyrna conceived a passion for her father, and with the complicity of her nurse she shared her father's bed without his knowledge for twelve nights. But when he was aware of it, he drew his sword and pursued her, and being overtaken she prayed to the gods that she might be invisible; so the gods in compassion turned her into the tree which they call* smyrna *(myrrh). Ten months afterwards the tree burst and Adonis, as he is called, was born, whom for the sake of his*

[3] The classical texts we used as examples come from the Perseus Digital Library at Tufts University (`http://www.perseus.tufts.edu/`).

beauty, while he was still an infant, Aphrodite hid in a chest unknown to the gods and entrusted to Persephone. But when Persephone beheld him, she would not give him back. The case being tried before Zeus, the year was divided into three parts, and the god ordained that Adonis should stay by himself for one part of the year, with Persephone for one part, and with Aphrodite for the remainder. However Adonis made over to Aphrodite his own share in addition; but afterwards in hunting he was gored and killed by a boar.

It must be possible to relate each of the terms in the CF consequently to stories such as these. In order to do this it is necessary to identify a set of dichotomies that can be consistently assigned according to the relationships in the CF. The two basic narrative characters, a and b must be identified in a consistent manner, with the added provision that the function y somehow transforms into an inversion of a (i.e. a^{-1}), and b to y, while x remains invariant under the chosen transformation.

Thus, for the above myth, an identification of the character Thias with the label b implies that the action of *killing* should be represented by x.[4] In order to proceed, we could hypothesise a scenario where the representation of the Adonis myth can be started with the following identification:

$F_y(b)$ as Thias "destroys" (in this case he kills Smyrna).

This move starts to limit the available identifications for the other variables in (1). Because the root of the CF in structuralism means that the assignments must be in binary opposition, we require a set of binary opposites for both the terms and the functions in this myth. The following are chosen for our current scenario:

Terms:	Functions:
– male/female	– affirm/deny
– divine/human	– active voice/passive voice
– adult/adolescent	– complete/incomplete

Thus, designating the male human adult Thias as b implies that $-b$ could represent a female human adult, while b^{-1} could be a male human who was adolescent (Adonis in this myth) etc. Essentially this value assignment is open

[4] The particular set of values we assigned to variables in the CF for this example was as follows: complete male/female: fertile/adult, incomplete male/female: infertile/adolescent; complete denial active voice: destroy/kill, complete affirmative active voice: procreate/bear, incomplete denial active voice: wound/hurt, incomplete affirmative active voice: heal; passive voice for the above: be destroyed/killed/begotten/born/wound/healed, plus the above being done either to the other or the self.

to a certain amount of freedom, yet once one binary value has been designated, its opposite must be interpreted for contrast in some manner. This inversion can be performed in one of the four following ways:

- a is a binary opposite of b
- a is a binary opposite of $-a$
- a is a binary opposite of a^{-1}.
- $x(a)$ is binary opposite of $a(x)$, here distinguishing between the other and the self.

The CF requires that each of these value assignments be performed consistently across the narrative. Continuing this process for the myth of Adonis, we can represent the full structure of the myth quoted above using the character map depicted in Table 1 and the function map in Table 2.

		complete?	
	male / female	yes	no
divine?	yes	a / $-a$	a^{-1} / $-a^{-1}$
	no	b / $-b$	b^{-1} / $-b^{-1}$

Table 1. A set of consistent value assignments for the characters in the myth of Adonis.

		affirm?	
	active voice / passive voice	yes	no (deny)
complete?	yes	x / $-x$	x^{-1} / $-x^{-1}$
	no	y / $-y$	y^{-1} / $-y^{-1}$

Table 2. A set of consistent function assignments for the myth of Adonis.

This set of mappings allows us to keep assigning variables to the narrative in the myth. Thus, we see a slightly symmetrical relationship between Thias and Adonis start to emerge within this narrative structure, which we can formalise using the item and function variables:

$F_y(b)$ as Thias (a male human adult) destroys someone else (in this case he kills Smyrna),

$F_x(b)$ as Thias creates someone else (i.e. begets Adonis, by sleeping with Smyrna),

$F_{a^{-1}}(y)$ as Adonis (a male adolescent divine) destroys himself (in this case he is killed by a boar but his wounds were obtained during a hunt in which he chose to participate).[5]

Finally, recalling the manner in which Aphrodite was born provides the final missing piece of the formula [11, lines 189–191]:

> *"And so soon as he had cut off the members with flint and cast them from the land into the surging sea, they were swept away over the main a long time: and a white foam spread around them from the immortal flesh, and in it there grew a maiden."*

Which leaves us with an understanding of $F_x(a)$ as the maiden that grew from the white foam that arose in that part of the sea where the genitals of Kronos' father (Uranos) landed:

$F_x(a)$**:** male divine adult creates someone else (in this case Uranos "creates" Aphrodite when his members were cast into the sea).

As contrasted with syntagms about divine or human adult males procreating and killing others, the crucial difference is the role the adolescent divine male who destroys himself. Thus, we see the final typical step emerge which distinguishes the CF from other possible narrative formulae. Note in particular the manner in which a double inversion of content takes place in the fourth argument: what used to be a term takes a reciprocal value, i.e. a maps to a^{-1}, and a former pair of functions and term values swap roles.

2.2 The narrative formula

A formula built from the same term vs. function value distribution but without the characteristic double inversion in the fourth argument is not a CF but something we will call a narrative formula (NF), to distinguish between them. An example would be

$$F_x(a) : F_y(b) :: F_x(b) : F_y(a). \quad (2)$$

[5] "It may be significant, however, that an accident in boar hunting [...] is liable to produce wounds somehow equalling castration; then the boar would be just an exchangeable sign for a deeper meaning" [10, p.108]. Castration as punishment or a voluntary act is frequent in the cult of a group of minor deities from Asia Minor, to which Adonis also belongs. Strictly speaking it is the boar who mutilates Adonis, not he himself, but as far as we know, on a higher level of abstraction these narrative elements belong together.

This is sometimes called the "weak" variant of the CF, i.e. its existence is acknowledged and explicated [12]. We note that this variant may be used to describe myths with a much more simple narrative structure, in particular, those that do not feature the characteristic double inversion of (1).

While the value sets of the four arguments are not defined but left to guesswork, based on suggested examples, the range of the CF spans from tribal myths [3] to Ancient Greek ones [13, 10] which would explain the *canonical* adjective attached to it. In spite of the claimed universal validity, its full potential is unexplored, partly going back to the fact that explanation attempts keep on working top down, i.e. trying to find phenomena which can be characterized by such dynamics.

The NF starts to provide a reason for the ongoing failure of the CF to be generally accepted as *the* mathematicalisation of mythology. Contrary to its name, the CF exists in several variants, partly suggested by Lévi-Strauss himself in different phases of his scholarly career, or by [14], [10], and [15]. This plethora of alternatives already hints at insecurities as to what exactly *the* CF might be, suggesting that perhaps it is just one valid variant among many [16, 8, 17].

2.3 How many narrative vs. canonical formulae are there?

It has been long suspected that not one but many forms of the CF exist, all pertinent to myth (and indeed, to several narrative genres). Here we introduce a consistent way to generate, and interpret, families of its variants. A more comprehensive approach to formula generation will have to be dealt with elsewhere. Three observations are pertinent here:

- There are three modifiers of term/function values in the NF and CF: the sign of the argument, the sign of the exponent, and the role swap between term and function values;
- Out of terms a, b and functions x and y plus one of the three modifiers per formula, one can create 4 x 8 = 32 "weak" forms of the CF (Table 3, left column). Typical for these is that although they may use one of the modified values, *there is no double inversion with respect to the relational structure of the group in them*;
- Not one but altogether 32 "strong" forms, including the original CF, can be formed by systematic *interaction* between two "weak" forms by exchanging the respective fourth arguments of NF_1 vs. NF_7, NF_2 vs. NF_8, NF_3 vs. NF_5, NF_4 vs. NF_6, NF_5 vs. NF_2, NF_6 vs. NF_1, NF_7 vs. NF_4, and NF_8 vs. NF_3 in the first octet of the collection of "weak" forms, respectively (Table 3). The rules of CF formation are similar for the other "weak" octets as well.

“Weak” forms	“Strong” forms
Octet A (term +, function +)	Octet E (term +, function +)
$\mathrm{NF}_1 = x(a) : y(b) :: x(b) : y(a)$	$\mathrm{CF}_1 = x(a) : y(b) :: x(b) : a^{-1}(y)$
$\mathrm{NF}_2 = x^{-1}(a) : y^{-1}(b) :: x^{-1}(b) : y^{-1}(a)$	$\mathrm{CF}_2 = x^{-1}(a) : y^{-1}(b) :: x^{-1}(b) : a^{-1}(y^{-1})$
$\mathrm{NF}_3 = x(a^{-1}) : y(b^{-1}) :: x(b^{-1}) : y(a^{-1})$	$\mathrm{CF}_3 = x(a^{-1}) : y(b^{-1}) :: x(b^{-1}) : a(y)$
$\mathrm{NF}_4 = x^{-1}(a^{-1}) : y^{-1}(b^{-1}) :: x^{-1}(b^{-1}) : y^{-1}(a^{-1})$	$\mathrm{CF}_4 = x^{-1}(a^{-1}) : y^{-1}(b^{-1}) :: x^{-1}(b^{-1}) : a(y^{-1})$
$\mathrm{NF}_5 = a(x) : b(y) :: b(x) : a(y)$	$\mathrm{CF}_5 = a(x) : b(y) :: b(x) : y^{-1}(a)$
$\mathrm{NF}_6 = a(x^{-1}) : b(y^{-1}) :: b(x^{-1}) : a(y^{-1})$	$\mathrm{CF}_6 = a(x^{-1}) : b(y^{-1}) :: b(x^{-1}) : y(a)$
$\mathrm{NF}_7 = a^{-1}(x) : b^{-1}(y) :: b^{-1}(x) : a^{-1}(y)$	$\mathrm{CF}_7 = a^{-1}(x) : b^{-1}(y) :: b^{-1}(x) : y^{-1}(a^{-1})$
$\mathrm{NF}_8 = a^{-1}(x^{-1}) : b^{-1}(y^{-1}) :: b^{-1}(x^{-1}) : a^{-1}(y^{-1})$	$\mathrm{CF}_8 = a^{-1}(x^{-1}) : b^{-1}(y^{-1}) :: b^{-1}(x^{-1}) : y(a^{-1})$
Octet B (term -, function +)	Octet F (term -, function +)
$\mathrm{NF}_9 = x(-a) : y(-b) :: x(-b) : y(-a)$	$\mathrm{CF}_9 = x(-a) : y(-b) :: x(-b) : -a^{-1}(y)$
$\mathrm{NF}_{10} = x^{-1}(-a) : y^{-1}(-b) :: x^{-1}(-b) : y^{-1}(-a)$	$\mathrm{CF}_{10} = x^{-1}(-a) : y^{-1}(-b) :: x^{-1}(-b) : -a^{-1}(y^{-1})$
$\mathrm{NF}_{11} = x(-a^{-1}) : y(-b^{-1}) :: x(-b^{-1}) : y(-a^{-1})$	$\mathrm{CF}_{11} = x(-a^{-1}) : y(-b^{-1}) :: x(-b^{-1}) : -a(y)$
$\mathrm{NF}_{12} = x^{-1}(-a^{-1}) : y^{-1}(-b^{-1}) :: x^{-1}(-b^{-1}) : y^{-1}(-a^{-1})$	$\mathrm{CF}_{12} = x^{-1}(-a^{-1}) : y^{-1}(-b^{-1}) :: x^{-1}(-b^{-1}) : -a(y^{-1})$
$\mathrm{NF}_{13} = -a(x) : -b(y) :: -b(x) : -a(y)$	$\mathrm{CF}_{13} = -a(x) : -b(y) :: -b(x) : y^{-1}(-a)$
$\mathrm{NF}_{14} = -a(x^{-1}) : -b(y^{-1}) :: -b(x^{-1}) : -a(y^{-1})$	$\mathrm{CF}_{14} = -a(x^{-1}) : -b(y^{-1}) :: -b(x^{-1}) : y(-a)$
$\mathrm{NF}_{15} = -a^{-1}(x) : -b^{-1}(y) :: -b^{-1}(x) : -a^{-1}(y)$	$\mathrm{CF}_{15} = -a^{-1}(x) : -b^{-1}(y) :: -b^{-1}(x) : y^{-1}(-a^{-1})$
$\mathrm{NF}_{16} = -a^{-1}(x^{-1}) : -b^{-1}(y^{-1}) :: -b^{-1}(x^{-1}) : -a^{-1}(y^{-1})$	$\mathrm{CF}_{16} = -a^{-1}(x^{-1}) : -b^{-1}(y^{-1}) :: -b^{-1}(x^{-1}) : y(-a^{-1})$
Octet C (term +, function -)	Octet G (term +, function -)
$\mathrm{NF}_{17} = -x(a) : -y(b) :: -x(b) : -y(a)$	$\mathrm{CF}_{17} = -x(a) : -y(b) :: -x(b) : a^{-1}(-y)$
$\mathrm{NF}_{18} = -x^{-1}(a) : -y^{-1}(b) :: -x^{-1}(b) : (-y^{-1}(a)$	$\mathrm{CF}_{18} = -x^{-1}(a) : -y^{-1}(b) :: -x^{-1}(b) : a^{-1}(-y^{-1})$
$\mathrm{NF}_{19} = -x(a^{-1}) : -y(b^{-1}) :: -x(b^{-1}) : a(-y)$	$\mathrm{CF}_{19} = -x(a^{-1}) : -y(b^{-1}) :: -x(b^{-1}) : a(-y)$
$\mathrm{NF}_{20} = -x^{-1}(a^{-1}) : -y^{-1}(b^{-1}) :: -x^{-1}(b^{-1}) : -y(a^{-1}$	$\mathrm{CF}_{20} = -x^{-1}(a^{-1}) : -y^{-1}(b^{-1}) :: -x^{-1}(b^{-1}) : a(-y^{-1})$
$\mathrm{NF}_{21} = a(-x) : b(-y) :: b(-x) : a(-y)$	$\mathrm{CF}_{21} = a(-x) : b(-y) :: b(-x) : -y^{-1}(a)$
$\mathrm{NF}_{22} = a(-x^{-1}) : b(-y^{-1}) :: b(-x^{-1}) : a(-y^{-1})$	$\mathrm{CF}_{22} = a(-x^{-1}) : b(-y^{-1}) :: b(-x^{-1}) : -y(a)$
$\mathrm{NF}_{23} = a^{-1}(-x) : b^{-1}(-y) :: b^{-1}(-x) : a^{-1}(-y)$	$\mathrm{CF}_{23} = a^{-1}(-x) : b^{-1}(-y) :: b^{-1}(-x) : -y^{-1}(a^{-1})$
$\mathrm{NF}_{24} = a^{-1}(-x^{-1}) : b^{-1}(-y^{-1}) :: b^{-1}(-x^{-1}) : a^{-1}(-y^{-1})$	$\mathrm{CF}_{24} = a^{-1}(-x^{-1}) : b^{-1}(-y^{-1}) :: b^{-1}(-x^{-1}) : -y(a^{-1})$
Octet D (term -, function -)	Octet H (term -, function -)
$\mathrm{NF}_{25} = -x(-a) : -y(-b) :: -x(-b) : -y(-a)$	$\mathrm{CF}_{25} = -x(-a) : -y(-b) :: -x(-b) : -a^{-1}(-y)$
$\mathrm{NF}_{26} = -x^{-1}(-a) : -y^{-1}(-b) :: -x^{-1}(-b) : -y^{-1}(-a)$	$\mathrm{CF}_{26} = -x^{-1}(-a) : -y^{-1}(-b) :: -x^{-1}(-b) : -a^{-1}(-y^{-1})$
$\mathrm{NF}_{27} = -x(-a^{-1}) : -y(-b^{-1}) :: -x(-b^{-1}) : -y(-a^{-1})$	$\mathrm{CF}_{27} = -x(-a^{-1}) : -y(-b^{-1}) :: -x(-b^{-1}) : -a(-y)$
$\mathrm{NF}_{28} = -x^{-1}(-a^{-1}) : -y^{-1}(-b^{-1}) :: -x^{-1}(-b^{-1}) : -y^{-1}(-a^{-1})$	$\mathrm{CF}_{28} = -x^{-1}(-a^{-1}) : -y^{-1}(-b^{-1}) :: -x^{-1}(-b^{-1}) : -a(-y^{-1})$
$\mathrm{NF}_{29} = -a(-x) : -b(-y) :: -b(-x) : -a(-y)$	$\mathrm{CF}_{29} = -a(-x) : -b(-y) :: -b(-x) : -y^{-1}(-a)$
$\mathrm{NF}_{30} = -a(-x^{-1}) : -b(-y^{-1}) :: -b(-x^{-1}) : -y(-a)$	$\mathrm{CF}_{30} = -a(-x^{-1}) : -b(-y^{-1}) :: -b(-x^{-1}) : -y(-a)$
$\mathrm{NF}_{31} = -a^{-1}(-x) : -b^{-1}(-y) :: -b^{-1}(-x) : -a^{-1}(-y)$	$\mathrm{CF}_{31} = -a^{-1}(-x) : -b^{-1}(-y) :: -b^{-1}(-x) : -y^{-1}(-a^{-1})$
$\mathrm{NF}_{32} = -a^{-1}(-x^{-1}) : -b^{-1}(-y^{-1}) :: -b^{-1}(-x^{-1}) : -b^{-1}(-y^{-1})$	$\mathrm{CF}_{32} = -a^{-1}(-x^{-1}) : -b^{-1}(-y^{-1}) :: -b^{-1}(-x^{-1}) : -y(-a^{-1})$

Table 3. By interaction between their respective fourth arguments, 32 CF can be generated from 32 NF.

All 31 new forms of the CF, i.e. $\mathrm{CF}_2 - \mathrm{CF}_{32}$ are functionally equivalent with CF_1 but stand for different semantic (conceptual) parameter combinations. In other words the CF as a narrative generation tool performs the same transformations on the plot but under rotation of its group, leading to new actors and actions in new situations. There are also ways to derive more NF variants which can describe increasingly complex mythological situations. A more generic probabilistic approach will be discussed in section 3.

3 The canonical formula and quantum interaction

The relation between the left hand side and the right hand side of the canonical formula can be treated as a transformation, that is, $F_x(a) : F_y(b) \mapsto F_x(b) : F_{a^{-1}}(y)$.

According to Morava, "Lévi-Strauss is describing a logical system in which truth-values lie in an algebraic system called a noncommutative group" [18, p.55]. The noncommutative group is identified as the quaternion group of order eight with the elements $Q = \{\pm 1, \pm i, \pm j, \pm k\}$, with the noncommutative product operation defined as $ij = k = -ji$, $jk = i = -kj$, $ki = j = -ik$, $ii = jj = kk = -1$, and $(-1)^2 = 1$.

Let us define an antiautomorphism $\lambda : Q \mapsto Q$ as $\lambda(i) = k$, $\lambda(j) = -i$, and $\lambda(k) = j$. Assigning $x \mapsto 1$, $a \mapsto i$, $y \mapsto j$, and $b \mapsto k$, this automorphism reproduces the canonical formula [8].

The Pauli matrices are a set of three 2×2 complex matrices which are Hermitian and unitary. They are:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Together with the identity matrix I, they form a basis for the real Hilbert space of 2×2 complex Hermitian matrices. Each Pauli matrix is related to an operator that corresponds to an observable describing the spin of a spin-$1/2$ particle, in each of the corresponding three spatial directions.

The real linear span of $\{I, i\sigma_x, i\sigma_y, i\sigma_z\}$ is isomorphic to the real algebra of quaternions H. The isomorphism from H to this set is given by the following map:

$$1 \mapsto I, \quad i \mapsto -i\sigma_x, \quad j \mapsto -i\sigma_y, \quad k \mapsto -i\sigma_z. \tag{3}$$

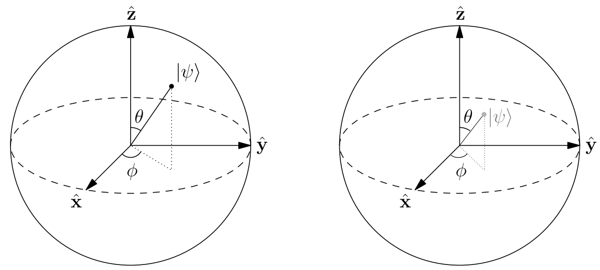
Since any 2×2 complex Hermitian matrices can be expressed in terms of the identity matrix and the Pauli matrices, 2×2 mixed states, that is, 2×2 positive semidefinite matrices with trace one, can be represented by the Bloch sphere. This can be seen by simply first writing a Hermitian matrix as a real linear combination of $\{I, \sigma_x, \sigma_y, \sigma_z\}$, then imposing the positive semidefinite and trace one assumptions. Thus a density matrix can be written as $\rho = \frac{1}{2}(I + s\sigma)$, where σ is a vector of the Pauli matrices, and s is called the Bloch vector. For pure states, this provides a one-to-one mapping to the surface of the Bloch sphere, and for mixed states, the Bloch vector lies in the interior of the Bloch ball.

Given the mapping between the canonical formula and the quaternion group of order eight, and in turn, the isomorphism between the real algebra of quaternions and the Pauli basis, we arrive at a probabilistic interpretation of the canonical formula, with a geometry provided by the Bloch sphere. Antiautomorphisms become rotations of pure or mixed states. This far with no apparent upper limit to construct NF, the syntagm occurrences and co-occurrences these match will not be equiprobable, neither will be the 4th arguments following identical tripartite initial strings, which in turn yields the probabilistic raw material the Pauli basis refers to.

Associating elements of the canonical formula with the Pauli matrices has a further advantage. As pointed out in sections 2.1 and 2.3, it is not necessarily obvious to give a well-cut interpretation of the CF, irrespective of whether we consider the weak or strong variants. Even versions of the same myth might elude interpretation. This is where a Pauli basis helps, where the weight of the basis correspond to probability values of the various components of the CF. Here we take probability values as a degree of belief, and we do not take a frequentist approach, although the latter might prove viable given a proper statistical analysis of myths and CF patterns. In pure states, the probability amplitudes must add to one, leading to a stricter, more formulaic reading of the CF. Mixed states, on the other hand, give full freedom in assigning probability values. In either case, weights might be chosen such that components of the CF are nullified. We believe that this mathematical description of the CF is more general than existing ones, and allows a lenient interpretation with a wider scope that may extend beyond myths.

4 Applying the probabilistic description

The fundamental difficulty with myths is "belief contamination," also called eclecticism or syncretism, i.e. different concepts belonging to the same category (e.g. the dying deity) can appear in the same plot so that nobody can tell them apart. The other one is the fundamental insecurity of not knowing what factor may be important and how much of its manifestations can be out there. E.g., what is the probability that a text fragment is in state $F_x(a)$, or a whole text as a mix of $F_x(a) : F_y(b) \mapsto F_x(b) : F_{a^{-1}}(y)$ has a given outcome for $F_{a^{-1}}(y)$? A probabilistic tool which, based on scalable text variant scanning, can indicate



(a) A pure state. (b) A mixed state.

Fig. 1. A pure state corresponds to a point on the surface of the Bloch sphere, whereas a mixed state is inside the Bloch sphere.

mixed vs. pure conceptual states and thereby answer such questions is something sorely missed. This is where QI can help.

Given that CF_1-CF_{32} correspond to pure state vectors on the surface of a Bloch sphere (Fig. 1(a)), whereas mixed states of a text appear as vectors pointing inside of the sphere, we tested our working hypothesis on a small corpus of thirteen texts from ancient Asia Minor, all concerned with Attis, a Phrygian dying god whose cult was imported to Rome as a consort of the Magna Mater, a variant of the Mediterranean Great Goddess. The plot is close to the Adonis myth: a youth either sacrifices his virility to the goddess or is punished by her to the same end. Out of the thirteen variants, in eight, Attis emasculates himself (direct self-mutilation); in one they mutually castrate each other with his partner (indirect self-mutilation); in two, he is either born as an eunuch or is killed by spear through an unspecified wound (indirect not-self mutiliation, i.e. killing by accident or similar); and in another two, it is the goddess who mutilates him (direct not-self mutilation). With $F_x(a)$ as the shift of the origin of the Bloch sphere standing for the beginning of the story, and the axes $F_y(b) = \hat{x}$, $F_x(b) = \hat{z}$, and $F_{a^{-1}}(y) = \hat{y}$, where the latter can have four outcomes as above, Fig. 1(b) shows the mixed state vector weighted by the outcome probabilities and the rest of the story alike.

5 Conclusions

We took a step toward bridging the gap between analytical studies in need of processing methodology vs. processing methodology development in need of raw material, by showing on a concrete example how a topical set of myth variants correspond via their syntagmatic transcripts to narrative formulae, i.e. formulaic expressions of condensed semantic content. The example came from fertility myths concerned with ritual punishment for wrongdoing as compensation underlying codified justice and community welfare regulation. We also demonstrated that there exist families of narrative formulae, some with double inverted values in their arguments, some without, which all share the same group structure with a certain quaternion group of order eight. Such formulae seem to be usable for information filtering. Beyond a group theoretic description, establishing a link to Pauli matrices, a quantum probabilistic framework further generalises the formulae.

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