

Application of spectral segregation index (SSI) as a measure of segregation at the individual level in peer networks

Mary-Anne Holfve-Sabel, mary-anne.holfve-sabel@hb.se School of Education and Behavioural Sciences, and Anders K.H. Bengtsson, anders.bengtsson@hb.se School of Engineering, University of Borås, Sweden.

Status group membership is important in everyday classrooms and during breaks. Students' classification may include rejected and controversial individuals (1). In classrooms popular or rejected groups have been shown to be stable over several years. Peer interaction conserves and exaggerates the cluster label (2). Classrooms without outliers indicate a lower risk of malaise (3). Furthermore the problem with marginalized individuals is related to whether individual students become discriminated. The situation with existing discrimination due to status, gender or ethnicity is closely related to segregation. The problem of segregation and discrimination was recently addressed in a report concerning member states of EU (4), which acknowledges segregation on school level. Considerable variation of segregation may occur between districts within a country (5). Gustafsson (6) has recently pointed out the need for investigation of school segregation in Sweden.

Thus, it is of great relevance to investigate aspects of segregation and exclusion in classrooms. The hypothesis is that an individual is more segregated the more the members of his/her group interact only with each other. The Spectral Segregation Index (SSI) provides a new measure of segregation and integration among groups of people based on quantitative data on interpersonal relationships. The index, pioneered by Echenique and Fryer (7), has been applied to large databases of different ethnical groups in the United States (7, 8). Theoretically the index is based on the Perron-Frobenius theorem on eigenvalues and eigenvectors of irreducible non-negative matrices in linear algebra, hence the reference to "spectral". Here we propose to use SSI to measure similar parameters among school children. To the best of our knowledge, studies with this method have not been conducted previously in Europe.

The special property of the SSI that makes it interesting for our purpose is its capability not only to measure integration/segregation for a group of people, such as a school class, but also to provide measures for individual pupils. This makes it possible to merge the data from the individual level with other kinds of individual data and draw conclusions concerning correlations between levels of integration/segregation and other characteristics of the individuals.

Aim

The aim was to solve some crucial methodological issues in order to demonstrate individual student relation with peers in different settings (work in classroom and during breaks) and from different perspectives (individual, interactive or group).

Design and methodology

Self reports on attitudes from 1540 students in 78 classes in grade 6 in Göteborg (9) were used. Each student was asked to write the first name (and the first initial of the last name) of the 3 peers he/she preferred to work with in the classroom, and preferred to play with during

breaks, in the order 1,2,3. Foreign or Nordic ethnicity was decided from the first name together with teacher report.

The data was quantified by assigning numerical weights to the choices, resulting in a, generically non-symmetric, weighted adjacency matrix of inter-personal strength of relationships. The sum of weights given to the 3 choices of a student should always be 1 (7).

Interpersonal relationships are directed. Between any two individuals there might be zero, one directed or relations from both directions, only the last one confirming mutual relationships. The students consist of individuals which 1) make choices and are chosen as well, 2) make choices but are not chosen (lonely), 3) are chosen but do not make choices (popular), 4) make no choices and are not chosen (outliers).

The mathematical conditions of the Perron-Frobenius theorem can be met by symmetrizing the adjacency matrix. The highest eigenvalue yields the integration measure for the group and the corresponding eigenvector provides the individual indices. The indices are computed using software such as MATLAB™.

Results:

Analysis of the networks in a classroom was done using algorithms written in MATLAB™. For each individual, the network is walked through in the direction of the relations and a list of visited members is collected in a report. Based on these reports the structure of the whole class can be worked out. It is indeed necessary to know if the class consists of 1, 2 or more clusters which have no connection with each other (i.e. are irreducible). SSI can only be computed for connected components of the network.

The adjacency matrix shaped by the program can be chosen in different ways: a) all choices are considered mutual, b) the perspective of the individual is the main objective, or c) the group perspective is the main objective.

In case any individual choice is considered mutual the matrix is symmetrised by using the mean of the weights between any two connected individuals. This gives a denser matrix where categories 2 and 3 above are pulled inside the main cluster. Intuitively, working with a symmetrised matrix, consolidates the network and gives softer values for the individual ssi. The matrix can also be transposed, which switches rows and columns. Intuitively transposing gives the whole group's view on the individuals.

Finally, the matrix can be semi-symmetrised. This is done by computing the geometric mean of the weights between any two connected individuals. If the weight is zero in one direction, the result is a zero weight in both directions. This fragmentises larger cluster into smaller networks. Individuals of categories 2 and 3 above are joining category 4 as outliers. Each small network represents close bonds.

It seems that transposed or semi-symmetrised matrices are useful for highlighting network structure. This is an area of research that seems not have been extensively studied.

Perron-Frobenius theorem for irreducible, non-negative matrices was used for computation.

The highest eigenvalue (σ) of the equation gives the SSI for the group. An individual (normalized) ssi is computed from the eigenvalue, the eigenvector \mathbf{s}_i (built upon all choices an individual gets from peers) and $|\mathbf{s}_i|$ (sum of the elements of the eigenvector \mathbf{s}_i) using:

$$\hat{s}_i = \frac{\sigma}{|s|} s_i.$$

:
The sum of the individual normalised ssi is the group SSI.

An example of the results using the different matrices on the same group is shown below.

An authentic example
19 students, two represent other ethnicity (no 18 & 22)

Type of matrix	Original	Symmetric	Transponed	Semi symmetric
Ind. outliers	no 22	no 22	11,15,22	7,8,11,15,19,22
No of clusters	2 (overlap)	1	2	3
Ind no 18 ssi	0.07	0.05	0.03	0.09
Highest ind ssi	0.07	0.10	0.17	0.29
Group SSI	0.99	1.08	1.00	0.77
Note	no 11 & 15 are here missing This matrix is abandoned	Here no 18 has about 5% choices a medium position	The group does not look upon no 18 as a very popular student	The smaller network including no 18 look rather positive on him as a work mate

The weights given to the three choices affect ssi. Obviously, there is a fair amount of arbitrariness in the assignment of weights. On the other hand, one can argue for a range of quite natural choices. By varying the weights we can investigate the stability of the indices to different choices of weights, and in that way gauge the strength of the subsequent conclusions. A reasonable solution for the first weight was seen around 0.5-0.6 (e g 0.6, 0.3, 0.1). When different weights were applied (0.5; 0.34; 0.16) on the 3 choices, individuals with high status emerged clearly. A highly appreciated student during work could have low popularity during breaks.

Conclusions so far

The individual ssi can be calculated and compared between individuals.

The choice of weights matter for the individual ssi.

The type of matrix used is highly important for the results. Here a transponed matrix seems promising for revealing the group perspective on the individual, while a semi-symmetric matrix highlights groups with tighter two-way bonds.

Further development of the algorithms of the programme is necessary. In the near future it will be possible to highlight for instance individual and group segregation. Other aspects of usefulness of the method are the possibility to compare gender patterns within different classrooms and patterns of interaction within classrooms with different proportions of “immigrants” and Nordic individuals. Furthermore, segregated clusters and rejected students can be visualized.

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